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DVCS amplitude with kinematical twist-3 terms

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We compute the amplitude of deeply virtual Compton scattering (DVCS) using the calculus of QCD string operators in coordinate representation. To restore the electromagnetic gauge invariance (transversality) of the twist-2 amplitude we include the operators of twist-3 which appear as total derivatives of twist-2 operators. Our results are equivalent to a Wandzura-Wilczek approximation for twist-3 skewed parton distributions. We find that this approximation gives a finite result for the amplitude of a longitudinally polarized virtual photon, while the amplitude for transverse polarization is divergent, *i.e.*, factorization breaks down in this term.

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I. INTRODUCTION

The deeply virtual Compton scattering (DVCS) process, in which a highly virtual photon $\gamma^*(q_1)$ produces a real photon $\gamma(q_2 = q_1 + r)$ at small invariant momentum transfer $t = r^2$, is receiving a lot of attention as a potential source of new information about the nucleon structure in terms of so-called "skewed" parton distributions (SPD's) [1–4]. The first experimental observation of DVCS has been already reported by the ZEUS collaboration [5]. Originally, only the purely twist–2 contribution to Compton amplitude was included [1,2], which violates electromagnetic (EM) gauge invariance in terms linear in the transverse part $r_\perp \equiv \Delta$ of the momentum transfer. To overcome this problem, Guichon and Vanderhaeghen [6] proposed to add an "ad hoc" $O(\Delta)$ term restoring the EM gauge invariance of the twist-2 DVCS amplitude. The correctness of their choice was recently confirmed by several groups [7–9] which derived this term in a regular way as a kinematical twist-3 contribution. Furthermore, Anikin *et al.* [7], using the momentum–space collinear expansion, have obtained expressions which include (in case of the pion target) all the relevant twist–3 operators. The DVCS amplitude to accuracy $1/\sqrt{-q_1^2}$ was also calculated by Penttinen *et al.* [8] in a parton model approach. Within the light-cone expansion framework, Belitsky and Müller [9] analyzed both quark and quark-gluon contributions and demonstrated that, to get a gauge invariant result up to terms of order t/q_1^2, it is sufficient to retain only the part of the twist–3 SPD's which is obtained by Wandzura–Wilczek (WW) type formulas from the twist–2 distributions. Recently, Kivel *et al.* [10] established that the WW-approximation expression for the transverse polarization of the virtual photon diverges. The mathematical aspects of twist decomposition were discussed by Blümlein, Robaschik [11], Geyer and Lazar [12].

Our goal is to analyze the DVCS amplitude within the QCD string operator approach of Balitsky and Braun [13], which proved to be a powerful tool to investigate the higher-twist effects. Here, we consider only the kinematical twist-3 terms. In this sense, our results are equivalent to the WW approximation. In addition to offering an alternative derivation of this approximation, we incorporate the formalism of double distributions [2,14] which provides a simple way of deriving relations between new SPD's. For instance, the fact that the WW approximation gives finite results for the amplitude for longitudinally polarized photon, but diverges in the case of transverse polarization, can be easily understood on the basis of the formulas relating these SPD's and the basic twist-2 DD's.

II. DVCS AMPLITUDE

Generalities. The virtual Compton scattering amplitude is derived from the correlation function

$$T_{\mu\nu} = i \int d^4x \int d^4y \, e^{-i(q_1x)+i(q_2y)} \langle p_2 | \mathrm{T}\{J_\mu(x) J_\nu(y)\} | p_1 \rangle \ , \tag{1}$$

where $J^\mu(x)$ is the electromagnetic current operator. Due to current conservation $\partial^\mu J_\mu(x) = 0$, this function is transverse with respect to the incoming and outgoing photon momenta: $q_1^\mu T_{\mu\nu} = 0$, $q_2^\nu T_{\mu\nu} = 0$. It is convenient to switch to symmetric variables $q = (q_1 + q_2)/2$, $r = q_2 - q_1$ and $p = (p_1 + p_2)/2$. Then, the transversality conditions convert into two relations

$$q^\mu T_{\{\mu\nu\}} = \frac{r^\mu}{2} T_{[\mu\nu]} \quad , \quad q^\mu T_{[\mu\nu]} = \frac{r^\mu}{2} T_{\{\mu\nu\}} \tag{2}$$

connecting the symmetric $T_{\{\mu\nu\}} \equiv (T_{\mu\nu} + T_{\nu\mu})/2$ and antisymmetric $T_{[\mu\nu]} \equiv (T_{\mu\nu} - T_{\nu\mu})/2$ parts of $T^{\mu\nu}$. In the $r = 0$ forward limit, the two relations decouple to give the DIS transversality conditions $q^\mu T_{\{\mu\nu\}} = 0$, $q^\mu T_{[\mu\nu]} = 0$.

In DVCS, the initial photon is in the Bjorken kinematics $\{-q_1^2 \to \infty, (p_1q_1) \to \infty, x_B \equiv -q_1^2/[2(p_1q_1)] \text{ fixed}\}$ and the final one is real $q_2^2 = 0$. Since $q_2^2 = q_1^2 + 2(q_1r) + t$, the momentum transfer r in this process should have a large component in the direction of p, with (rq_1) close to $-q_1^2/2$ for small t. The size of this component is characterized by the skewedness parameter $\eta \equiv (rq)/2(pq)$. For DVCS, η coincides, up to $O(t)$ terms, with the generalized Bjorken variable $\xi \equiv -q^2/2(pq)$. Hence, the momentum transfer may be split into the component parallel to p and the remainder Δ

$$r = 2\xi p + \Delta \ , \tag{3}$$

which in the $t = 0$ limit is transverse both to p and q: $(\Delta q) = -t/4$, $(\Delta p) = -\xi t/2$. For finite t, all the components of Δ are of order $|t|^{1/2}$: $\Delta \propto |t|^{1/2}$.

Coordinate representation. An efficient way to study the behavior of Compton amplitudes in the Bjorken limit is to use the light–cone expansion for the product $\Pi_{\mu\nu}(x,y) \equiv i\mathrm{T} J_\mu(x) J_\nu(y)$ of two vector currents in the coordinate representation. Following Balitsky and Braun [13], we start from the formal light–cone expansion in terms of QCD string operators (with gauge links along the straight line between the fields which, for brevity, we do not write explicitly). The leading light–cone singularity is contained in the "handbag" contribution

$$\Pi_{\mu\nu}(z\,|\,X) = \frac{4iz_\rho}{\pi^2 z^4}\left\{ s_{\mu\rho\nu\sigma}\mathcal{O}_\sigma(z\,|\,X) - \epsilon_{\mu\rho\nu\sigma}\mathcal{O}_{5\sigma}(z\,|\,X)\right\}\,, \tag{4}$$

where $s_{\mu\rho\nu\sigma} = g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\nu\rho}$, $\quad X = (x+y)/2$, $z = y - x$, and

$$\begin{aligned}\mathcal{O}_\sigma(z\,|\,X) &= \frac{1}{2i}\,\left[\bar\psi(X - z/2)\gamma_\sigma\psi(X + z/2)\;-\;(z \to -z)\right]\,,\\ \mathcal{O}_{5\sigma}(z\,|\,X) &= \frac{1}{2}\,\left[\bar\psi(X - z/2)\gamma_\sigma\psi(X + z/2)\;+\;(z \to -z)\right]\,.\end{aligned} \tag{5}$$

Twist–2 part. The string operators in Eq. (4) do not have a definite twist. The twist–2 part is defined by formally Taylor–expanding the string operators in Eq. (4) in the relative coordinate z and retaining only the totally symmetric traceless parts of the coefficients in the expansion:

$$\left[\bar\psi(X - z/2)\gamma_\sigma\psi(X + z/2)\right]^{\mathrm{twist-2}} \equiv \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\mu_1}\ldots z_{\mu_n}\,\bar\psi(X)\left[\gamma_{\{\sigma}\,\overset{\leftrightarrow}{D}_{\mu_1}\ldots\overset{\leftrightarrow}{D}_{\mu_n\}}\,-\mathrm{traces}\right]\psi(X), \tag{6}$$

and similarly for the operator with $\gamma_\sigma\gamma_5$. As shown in Ref. [13], "symmetrization" and "subtraction of traces" can be carried out directly at the level of non-local operators. The part of the string operator corresponding to totally symmetric local tensor operators is projected out by

$$\left[\bar\psi(X - z/2)\gamma_\sigma\psi(X + z/2)\right]^{\mathrm{sym}} \;=\; \frac{\partial}{\partial z_\sigma}\int_0^1 dt\;\bar\psi(X - tz/2)\hat z\psi(X + tz/2) \tag{7}$$

(we use the notation $\hat z \equiv z^\sigma\gamma_\sigma$). The subtraction of traces in the local operators implies that the twist–2 string operator contracted with z_σ should satisfy the d'Alembert equation with respect to z:

$$\Box_z\left[\bar\psi(X - tz/2)\hat z\psi(X + tz/2)\right]^{\mathrm{twist-2}} = 0. \tag{8}$$

Transversality and twist–3 operators. In the coordinates X and z, the transversality conditions (2) are

$$\frac{\partial}{\partial z_\mu}\Pi_{\{\mu\nu\}}(z|X) = \frac{1}{2}\frac{\partial}{\partial X_\mu}\Pi_{[\mu\nu]}(z|X)\quad,\quad \frac{\partial}{\partial z_\mu}\Pi_{[\mu\nu]}(z|X) = \frac{1}{2}\frac{\partial}{\partial X_\mu}\Pi_{\{\mu\nu\}}(z|X)\,. \tag{9}$$

Consider the part of the current product given by Eq. (4) with the string operators replaced by their twist–2 parts. From Eq. (8) and $(\partial/\partial z_\rho)[z_\rho/(2\pi^2 z^4)] = -i\delta^{(4)}(z)$ it follows that $(\partial/\partial z_\mu)\,\Pi^{\mathrm{twist-2}}_{\{\mu\nu\}} = 0$, $(\partial/\partial z_\mu)\,\Pi^{\mathrm{twist-2}}_{[\mu\nu]} = 0$. Since forward matrix elements are zero for all total derivative operators, this guarantees the transversality of the twist–2 contribution in the case of deep inelastic scattering. In the non-forward case, we have $(\partial/\partial X_\mu)\Pi^{\mathrm{twist-2}}_{\{\mu\nu\}} \neq 0$, $(\partial/\partial X_\mu)\Pi^{\mathrm{twist-2}}_{[\mu\nu]} \neq 0$, and (9) is violated. The non-transverse terms in the twist–2 contribution can only be compensated by contributions from operators of higher twist. In fact, the necessary operators are contained in the part of the string operator which was dropped in taking the twist–2 part. Incorporating QCD equations of motion, it is possible to show [13] that the twist>2 part involves the total derivatives of string operators

$$\bar\psi(-z/2)\gamma_\alpha\psi(z/2)\;-\;\left[\bar\psi(-z/2)\gamma_\alpha\psi(z/2)\right]^{\mathrm{sym}} = \frac{i}{2}\epsilon_{\alpha\xi\rho\kappa}z_\xi\frac{\partial}{\partial X_\rho}\int_0^1 dt\,t\;\bar\psi(-tz/2)\gamma_\kappa\gamma_5\psi(tz/2) + \ldots\;\;. \tag{10}$$

The ellipses stand for quark–gluon operators (we do not write them explicitly since they are not needed to restore transversality of the twist–2 contribution). The relation for the operator with Dirac matrix $\gamma_\alpha\gamma_5$ is obtained by changing $\gamma_\alpha \to \gamma_\alpha\gamma_5$, $\gamma_\kappa\gamma_5 \to \gamma_\kappa$. The operators appearing under the total derivative on the R.H.S. of Eq. (10) and its $\gamma_\alpha\gamma_5$ analog are still the full string operators with no definite twist. Hence, one can decompose them into a symmetric (*i.e.*, twist–2) part and total derivatives, and so on; thus expressing the original string operator as the sum of its symmetric part and an infinite series of arbitrary order total derivatives of symmetric operators. This series can be summed up in a closed form (the details of the calculation are presented elsewhere [15]; similar expressions were derived independently in [9,10]):

$$\bar\psi(-z/2)\gamma_\sigma\psi(z/2) = \int_0^1 dv\ \left\{\cos\left[\frac{i\bar v}{2}\ \left(z\frac{\partial}{\partial X}\right)\right]\frac{\partial}{\partial z_\sigma}\ +\ \frac{iv}{2}\sin\left[\frac{i\bar v}{2}\ \left(z\frac{\partial}{\partial X}\right)\right]\frac{\partial}{\partial X_\sigma}\right\}\bar\psi(-vz/2)\hat z\psi(vz/2)$$
$$+\frac{i}{2}\epsilon_{\sigma\alpha\beta\gamma}z_\alpha\frac{\partial}{\partial X_\beta}\frac{\partial}{\partial z_\gamma}\int_0^1 dv\ \int_v^1 du\cos\left[\frac{i\bar u}{2}\ \left(z\frac{\partial}{\partial X}\right)\right]\ \bar\psi(-vz/2)\hat z\gamma_5\psi(vz/2)+\dots\ . \qquad (11)$$

An analogous formula applies to the operators with $\gamma_\sigma \to \gamma_\sigma\gamma_5$; one should just replace $\hat z \to \hat z\gamma_5,\ \hat z\gamma_5 \to \hat z$.

III. PARAMETRIZATION OF NONFORWARD MATRIX ELEMENTS

Double distributions. To get the amplitude for deeply virtual Compton scattering off a hadronic target we need parametrizations of the hadronic matrix elements of the uncontracted twist–2 string operators $\mathcal{O}_\sigma, \mathcal{O}_{5\sigma}$ appearing in Eq. (4). We will derive them from Eq. (11). For simplicity, we consider here one quark flavor and the pion target, which has zero spin and practically vanishing mass. In this case, the matrix element of the contracted *axial* operator $z^\sigma\mathcal{O}_{5\sigma}(z\,|\,0)$ (parametrized in the forward limit by the polarized parton density) is identically zero. Thus we need only the parametrization for the matrix element $\langle p-r/2\,|\,\mathcal{O}(z\,|\,0)\,|\,p+r/2\rangle$ of the contracted *vector* operator $\mathcal{O}(z\,|\,0) \equiv z^\sigma\mathcal{O}_\sigma(z\,|\,0)$. With respect to z, it can be regarded as a function of three invariants $(pz), (rz)$ and z^2. For dimensional reasons, the dependence on z^2 is through the combinations tz^2 and p^2z^2 only. Since we are going to drop $O(t)$ and $O(p^2)$ terms in the Compton amplitude, we ignore the dependence on z^2 and treat this matrix element as a function of just two variables (pz) and (rz). Incorporating the spectral properties of nonforward matrix elements [14] , we write the plane wave expansion in the form

$$\langle p-r/2\,|\,\mathcal{O}(z\,|\,0)\,|\,p+r/2\rangle = 2(pz)\int_{-1}^1 d\tilde x\int_{-1+|\tilde x|}^{1-|\tilde x|} e^{-i(kz)}f(\tilde x,\alpha)\,d\alpha + (rz)\int_{-1}^1 e^{-i\alpha(rz)/2}\,D(\alpha)\,d\alpha\ , \qquad (12)$$

where $k = \tilde x p + \alpha r/2$, $f(\tilde x,\alpha)$ is the double distribution (DD) and $D(\alpha)$ is the Polyakov-Weiss (PW) distribution amplitude [16] absorbing the (pz)-independent terms. From this parametrization, we can obtain the matrix elements of original uncontracted string operators, (11), including the kinematical twist–3 contributions. We consider first the part coming from the double distribution term in Eq. (12); the contributions from the PW–term will be included separately. In matrix elements, the total derivative turns into the momentum transfer, $i\partial/\partial X_\sigma \to r_\sigma = 2\xi p_\sigma + \Delta_\sigma$. Similarly, we write $k = (\tilde x + \xi\alpha)p + \alpha\Delta/2$. Expanding up to terms linear in the transverse momentum Δ we get

$$\frac{1}{2}\,\langle p-r/2\,|\,\mathcal{O}_\sigma(z\,|\,0)\,|\,p+r/2\rangle = \int_{-1}^1 d\tilde x\int_{-1+|\tilde x|}^{1-|\tilde x|} d\alpha\, f(\tilde x,\alpha)\left\{e^{-i(\tilde x+\xi\alpha)(pz)}p_\sigma\left[1-i\,\frac{\alpha}{2}\,(\Delta z)\right]\right.$$
$$\left.+\frac{1}{2}\left[\Delta_\sigma(pz)-p_\sigma(\Delta z)\right]\int_0^1 dv\, v\, e^{-iv(\tilde x+\xi\alpha)(pz)}\left[\sin(\bar v\xi(pz)) - i\alpha\cos(\bar v\xi(pz))\right]\right\}\ . \qquad (13)$$

Skewed distributions. The spectral parameter $\tilde x$ appears in Eq. (13) only in the combination $x \equiv \tilde x + \xi\alpha$, so we can introduce two skewed parton distributions:

$$\left.\begin{matrix} H(x,\xi) \\ A(x,\xi)\end{matrix}\right\}\ \equiv\ \int_{-1}^1 d\tilde x\int_{-1+|\tilde x|}^{1-|\tilde x|} d\alpha\,\delta(x-\tilde x-\xi\alpha)\,f(\tilde x,\alpha)\ \left\{\begin{matrix} 1 \\ \alpha\end{matrix}\right. \qquad (14)$$

Note that, in our case, the DD $f(\tilde{x},\alpha)$ is even in α and odd in $\tilde{x}$. As a result, the functions H and A satisfy the symmetry relations

$$H(x,\xi) = -H(-x,\xi) \quad , \quad H(x,\xi) = H(x,-\xi) \quad , \quad A(x,\xi) = A(-x,\xi) \quad , \quad A(x,\xi) = -A(x,-\xi). \tag{15}$$

Furthermore, because of the antisymmetry of the combination $\alpha f(\tilde{x},\alpha)$ with respect both to x and α we have

$$\int_0^1 dx\, A(x,\xi) \;=\; 0. \tag{16}$$

Hence, the distribution $A(x,\xi)$ cannot be a positive-definite function on $0 \le x \le 1$.

Combining the cosine and sine functions with the overall exponential factor, $e^{-ivx(pz)}$, one gets $vx \pm \bar{v}\xi$ combinations. Using (15), one can arrange that only $vx + \bar{v}\xi$ would appear:

$$\begin{aligned}\frac{1}{2}\,\langle p - r/2\,|\,\mathcal{O}_\sigma(z\,|\,0)\,|\,p + r/2\rangle &= p_\sigma \int_{-1}^{1} dx\, e^{-ix(pz)} \left[H(x,\xi) - \frac{i(\Delta z)}{2} A(x,\xi)\right] \\ + \frac{i}{2}\left[\Delta_\sigma(pz) - p_\sigma(\Delta z)\right] &\int_{-1}^{1} dx\,\left[H(x,\xi) - A(x,\xi)\right] \int_0^1 dv\, v\, \cos[(vx + \bar{v}\xi)(pz)]\ .\end{aligned} \tag{17}$$

In a similar fashion, we parametrize the matrix element of the axial string operator (11):

$$\frac{1}{2}\,\langle p - r/2\,|\,\mathcal{O}_{5\sigma}(z\,|\,0)\,|\,p + r/2\rangle = \frac{i}{2}\,\epsilon_{\sigma\alpha\beta\gamma}\, z_\alpha \Delta_\beta p_\gamma \int_{-1}^{1} dx\,\left[H(x,\xi) - A(x,\xi)\right] \int_0^1 dv\, v\, \sin[(vx + \bar{v}\xi)(pz)]\ . \tag{18}$$

Note that it is expressed in terms of the same skewed distributions $H(x,\xi)$ and $A(x,\xi)$ which, in turn, are determined by the original double distribution $f(\tilde{x},\alpha)$, see Eq. (14).

IV. DVCS AMPLITUDE FOR PION TARGET

DD-generated contribution. Substituting the parametrizations (17) and (18) into Eq. (4) and performing the Fourier integral over the separation z we obtain the Compton amplitude

$$\begin{aligned}T_{\mu\nu} = \;& \frac{1}{(pq)}\left[p_\mu q_\nu + q_\mu p_\nu - g_{\mu\nu}(pq) + 2\xi p_\mu p_\nu - \frac{\Delta_\nu}{2} p_\mu + p_\nu \frac{\Delta_\mu}{2}\right] \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i0} \\ &+ \frac{1}{2(pq)} \int_{-1}^{1} dx\, R(x,\xi) \int_0^1 dv\, \frac{\Delta_\nu(q_\mu + 3\xi p_\mu)}{\xi + vx + \bar{v}\xi - i0} + \frac{1}{2(pq)} \int_{-1}^{1} dx\, R(x,\xi) \int_0^1 dv\, \frac{(q_\nu + \xi p_\nu)\Delta_\mu}{-\xi + vx + \bar{v}\xi + i0}\ ,\end{aligned} \tag{19}$$

where $R(x,\xi)$ is the universal SPD describing the kinematical twist-3 contributions:

$$R(x,\xi) \equiv \frac{\partial H(x,\xi)}{\partial x} - \frac{\partial A(x,\xi)}{\partial x}\ . \tag{20}$$

This result for the Compton amplitude contains the same tensor structures as those obtained in Refs. [7,8]. All three terms in Eq. (19) are individually transverse up to terms of order t.

Singularities. The first term is the twist–2 part with the tensor structure corrected exactly as suggested by Guichon and Vanderhaeghen [6]. The integral over x exists if $H(x,\xi)$ is continuous at $x = \xi$, which is the case for SPD's derived from the DD's which are less singular than $1/\tilde{x}^2$ for $\tilde{x} = 0$ and are continuous otherwise (see [17]). In particular, continuous SPD's were obtained in model calculations of SPD's at a low scale in the instanton vacuum [18]. The second term contributes only to the helicity amplitude for a longitudinally polarized initial photon. The parameter integral over v gives the function $[\ln(x + \xi - i0) - \ln(2\xi - i0)]/(x - \xi)$ which is regular at $x = \xi$ and has a logarithmic singularity at $x = -\xi$. The integral over x exists if $R(x,\xi)$ is bounded at $x = -\xi$, which again is the case in the DD-based models described in Ref. [17]. The third term of Eq. (19) corresponds to the transverse polarization of the initial photon. In this case, one faces the integrand $1/[v(x-\xi)+i0]$ which produces dv/v divergence for the v-integral at the lower limit. One may hope to get a finite result only if the integral

$$I(\xi) \equiv \int_{-1}^{1} dx \frac{R(x,\xi)}{x-\xi+i0} \tag{21}$$

vanishes. From the definition of the skewed distributions $H(x,\xi)$ and $A(x,\xi)$ (14) it follows that

$$\frac{\partial A(x,\xi)}{\partial x} = -\frac{\partial H(x,\xi)}{\partial \xi}.$$

Hence, one can substitute $R(x,\xi)$ by the combination $\partial H(x,\xi)/\partial x + \partial H(x,\xi)/\partial \xi$ similar to that used in [10] within the context of WW approximation (in Refs. [9,10], full SPD's are implied while our $H(x,\xi)$ does not include the PW term). By analogy with Ref. [10], we integrate the $\partial H(x,\xi)/\partial x$ term by parts. This gives

$$I(\xi) = \frac{d}{d\xi} \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i0} \ ,$$

i.e., the ξ derivative of the twist-2 contribution. In general, the latter has a nontrivial ξ-dependent form determined by the shape of SPDs (see, however, the discussion of the PW contribution below). We conclude that the twist-3 amplitude for the transverse polarization of the initial photon diverges. A similar observation has been recently made in Ref. [10]. Since this amplitude is power–suppressed by a factor of $1/q^2$, factorization for DVCS works up to (and including) $1/\sqrt{-q^2}$–contributions to observables. Still, this is sufficient to allow for experimental tests of QCD predictions for DVCS.

WW-type representation. We can express our results in another form, introducing new skewed distributions related to $R(x,\xi)$ *via* an integral transformation similar to that used by Wandzura and Wilczek [19] * . This allows us to establish the equivalence between our approach and the WW-type approximation proposed in Ref. [9]. Treating the combination $xv+\bar{v}\xi$ in (19) as a new variable we define

$$R_W(x,\xi) \equiv \int_{-1}^{1} R(y,\xi)\, dy \int_0^1 \delta(yv+\bar{v}\xi - x)\, dv \ = \ \theta(x\geq\xi) \int_x^1 \frac{R(y,\xi)}{y-\xi}\, dy - \ \theta(x\leq\xi)\int_{-1}^{x} \frac{R(y,\xi)}{y-\xi}\, dy \ . \tag{22}$$

In terms of this transform, the matrix element of the vector operator (17) can be expressed as

$$\begin{aligned}\frac{1}{2}\langle\, p-r/2\,|\, \mathcal{O}_\sigma(z\,|\,0)\,|\, p+r/2\rangle = \int_{-1}^{1} dx\, e^{-ix(pz)} \Bigg\{ & p_\sigma H(x,\xi) - \frac{i}{2}\, p_\sigma\, (\Delta z)\, A(x,\xi) \\ & + \frac{1}{4}\left(\Delta_\sigma - p_\sigma \frac{(\Delta z)}{(pz)}\right)\Big[R_W(x,\xi) - R_W(-x,\xi)\Big]\Bigg\} \ .\end{aligned} \tag{23}$$

Note that only the odd part of $R_W(x,\xi)$ contributes here. In case of the axial operator (18)

$$\frac{1}{2}\langle\, p-r/2\,|\, \mathcal{O}_{5\sigma}(z\,|\,0)\,|\, p+r/2\rangle = \frac{i}{4}\,\epsilon_{\sigma\alpha\beta\gamma}\frac{z_\alpha}{(pz)}\Delta_\beta p_\gamma \int_{-1}^{1} dx\, e^{-ix(pz)}\Big[R_W(x,\xi) + R_W(-x,\xi)\Big] \tag{24}$$

only the even part of $R_W(x,\xi)$ appears. The part of the Compton amplitude (19) containing $R(x,\xi)$ can be written in terms of this new function as

$$\frac{1}{2(Qp)}\int_{-1}^{1}\left[\frac{\Delta^\mu(q^\nu+\xi p^\nu)}{x-\xi+i0} + \frac{\Delta^\nu(q^\mu+3\xi p^\mu)}{x+\xi-i0}\right] R_W(x,\xi)\, dx \ . \tag{25}$$

The integrals with $1/(x\pm\xi\mp i0)$ converge only if the function $R_W(x,\xi)$ is continuous for $x=\pm\xi$. According to Eq. (22), $R_W(x,\xi)$ is given by the integral of $R(y,\xi)/(y-\xi)$ from x to 1 if $x>\xi$ and from x to -1 if $x<\xi$. Evidently,

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*WW-type integrals of parton distributions have originally appeared within the ξ-scaling formalism [20] .

$x = -\xi$ is not a special point in the integral transformation (22), hence the function $R_W(x,\xi)$ is continuous at $x = -\xi$. However, it is extremely unlikely that the limiting values approached by $R_W(x,\xi)$ for $x = \xi$ from below and from above do coincide. Indeed, the difference of the two limits can be written as the principal value integral (compare with [10])

$$R_W(\xi+0,\,\xi) - R_W(\xi-0,\,\xi) = \mathrm{P}\int_{-1}^{1} \frac{R(y,\xi)}{y-\xi}\,dy\;, \tag{26}$$

which can be converted into the ξ-derivative of the real part of the twist–2 contribution. This means that the singularity, which we observed as a straight divergence of the dv/v integral, in this approach appears due to an unavoidable discontinuity of the $R_W(x,\xi)$ transform at $x = \xi$.

Contribution from the PW–term. The contribution of the PW term to the vector operator

$$\frac{1}{2}\,\langle p-r/2\,|\,\mathcal{O}_\sigma(0\,|\,z)\,|\,p+r/2\rangle_{\text{PW–Term}} \;=\; \frac{r_\sigma}{2}\int_{-1}^{1} d\alpha\, e^{-i\alpha(rz)/2}\,D(\alpha) \tag{27}$$

has a simple structure corresponding to a parton picture in which the partons carry the fractions $(1\pm\alpha)/2$ of the momentum transfer r. Since only one momentum r is involved, this term can contribute only to the totally symmetric part of the vector string operator: it "decouples" in the reduction relations (10). In particular, the PW term does not contribute to the second contribution in Eq. (11) which is generated by decomposition of the axial string operator: both derivatives, with respect to X and z, give rise to the momentum transfer r, whence the contraction with the ϵ–tensor in (11) gives zero. Thus, the PW-contribution should be transverse by itself. Indeed, a straightforward calculation gives

$$T_{\mu\nu}|_{\rm PW} \;=\; \frac{1}{(rq)}\Big[r_\mu q_\nu + q_\mu r_\nu - g_{\mu\nu}(rq) + r_\mu r_\nu\Big]\int_{-1}^{1}\frac{D(\alpha)}{\alpha-1}\,d\alpha\;, \tag{28}$$

which evidently satisfies $q_\mu T_{\mu\nu}|_{\rm PW} = 0,\; r_\mu T_{\mu\nu}|_{\rm PW} = 0$. Hence, this term can be treated as a separate contribution.

Alternatively, one may include it into the basic SPD $H(x,\xi)$ and all SPD's derived from $H(x,\xi)$. Specifically, for $\xi > 0$, the PW contribution to $H(x,\xi)$ is $D(x/\xi)\,\theta(|x|\le\xi)$ [16]; it contributes $(\xi-x)D'(x/\xi)\,\theta(|x|\le\xi)/\xi^2$ [where $D'(\alpha)\equiv(d/d\alpha)D(\alpha)$] to $R(x,\xi)$; furthermore, the PW contribution to $R_W(x,\xi)$ is $D(x/\xi)\,\theta(|x|\le\xi)/\xi$. Inserting these functions into Eqs. (19) and (25) one rederives Eq. (28). One can also observe that the PW term gives zero contribution into $I(\xi)$, Eq. (21).

V. CONCLUSIONS

In this paper, we have studied the DVCS amplitude making use of the light–cone expansion in terms of QCD string operators in the coordinate space. We have demonstrated that transversality of the light–cone expansion can be maintained by including a minimal set of "kinematical" twist–3 operators, which appear as total derivatives of twist–2 operators. Incorporating the formalism of double distributions, we established that the kinematical twist–3 contributions are described by a universal skewed parton distribution $R_W(x,\xi)$ which can be derived from the basic twist–2 double distribution $f(\tilde{x},\alpha)$. The new SPD $R_W(x,\xi)$ has the structure of a generalized WW transform. Due to discontinuities of $R_W(x,\xi)$ for $x = \xi$, the factorization for DVCS breaks down at the twist-3 level for the part of the amplitude corresponding to the transverse virtual photon. Physically, this happens when the virtual quark connecting the photon vertices becomes real and has the momentum practically coinciding with the momentum of the final photon. Such a quark cannot be treated as "hard", and the relevant contribution is similar to the soft contribution (Feynman mechanism) for hadronic form factors. The final photon is described then by the photon wave function rather than by a pointlike vertex. Such a separation of quark virtualities into hard and soft parts brings in a factorization scale μ, and would one get a $\ln(Q^2/\mu^2)$ factor instead of a logarithmic divergence. The study of the soft/hard interplay for higher-twist contributions in DVCS is an interesting and practically important problem for future studies. A promising approach is provided by the light-cone QCD sum rules [21].

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